

The simple pendulum

THEORY

The equation of motion of a small object m suspended from the end of a light, inextensible string of length L swinging in a vertical plane is the following:

$$m \cdot a_t = m \cdot L \frac{d^2 \alpha}{dt^2} = -m \cdot g \cdot \sin \alpha$$

where α is the time dependent angle of the string measured from the equilibrium (vertical) position. As it is a nonlinear differential equation we introduce the following approximation:

$$\sin \alpha \approx \alpha$$

and so the problem is similar to that of the spring:

$$m \cdot L \frac{d^2 \alpha}{dt^2} = -m \cdot g \cdot \alpha \quad \rightarrow \quad \frac{d^2 \alpha}{dt^2} = -(g/L) \cdot \alpha$$

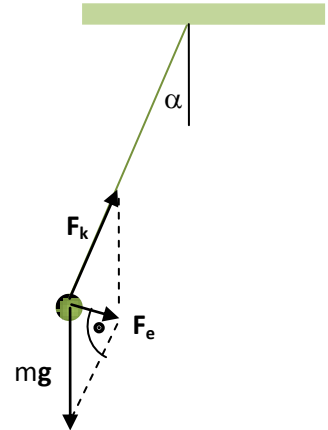
$$\alpha(t) = \alpha_{\max} \cos(\omega t + \varphi_0) \quad \rightarrow \quad \frac{d^2 \alpha}{dt^2} = -\omega^2 \cdot \alpha$$

$$\rightarrow \quad \omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{T}$$

This means that the motion of the pendulum is harmonic with a time period of

$$T = 2\pi \sqrt{\frac{L}{g}}.$$

The deviation caused by the approximation $\sin \alpha \approx \alpha$ is 0.05% for $\alpha = 5^\circ$, 1% for $\alpha = 22^\circ$ and 18% for $\alpha = 90^\circ$.



MEASUREMENT

Set the pendulum bob in motion with a small initial angle.

Measure the time of 10 periods.

Repeat the measurement 5 times.

Measure the length of the string L .

Measure the time of 10 periods when the initial angle is increased.

EVALUATION – HOMEWORK

1. Calculate the average time period \bar{T} and the confidence interval ΔT for $P = 95\%$.
2. Calculate the mean value of the acceleration of gravity \bar{g} (using the value \bar{T}).
3. Calculate the confidence interval Δg using the formula of error propagation.