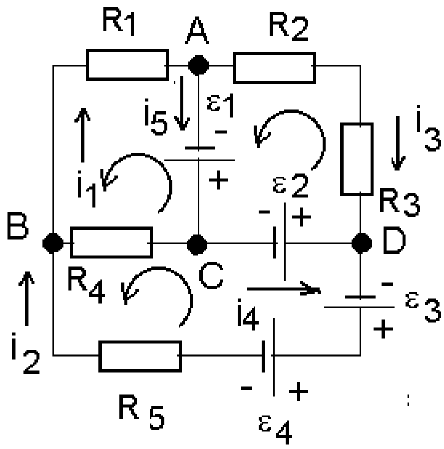
Electrical circuits



A more complicated ***electrical circuit*** can contain several sources and resistors and other devices, all connected with metal wires or strips. We will neglect the resistance of the connecting wires with respect to the other resistances in the circuit.

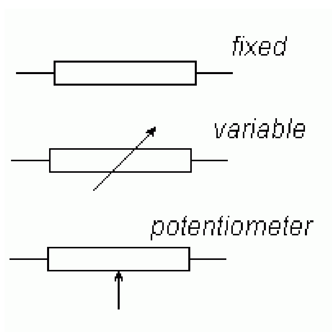
A circuit can contain several ***loops****:* these are non-crossing closed paths. There are ***junctions (nodes)*** like A, B, C, D, where several wires join. The part of the circuit between two neighbouring junction is called ***branch***. In a branch, the same current flows through each element.

Kirchhoff's laws in electrical circuits:

Kirchhoff's first law for junctions
As the electric charge is conserved, and the charge can not accumulate at a junction, the sum of the currents entering any junction must be equal to the sum of the currents leaving the junction. Using positive sign with the entering current and negative sine with the leaving currents, **the net current at a junction must be zero**.

Kirchhoff's second law for loops
The electric force is conservative in a static or stationary electric field, the work of the field along a closed path is zero. The same holds for a DC electric circuit. It is possible to assign a potential value (U) to each point of a circuit and the work done by the electric field on a positive unit charge when it moves from point A to B is equal to the difference of the potentials U(A)-U(B). **Along a closed loop, the sum of all potential differences is zero.**

Resistors are parts of electric circuits with defined resistance ranging from about 1Ω to several hundred MΩ. They are made from rods of a poor conductor (carbon grains mixed with insulating resin), or of resistance wire of or a thin strip of metal wound around some insulating support. The symbols for resistors in a circuit diagram are shown in the picture.
The potentiometer is a three-terminal device with a sliding contact as the third contact. The resistance between one end the sliding contact can be varied.



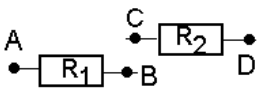
Ohm's Law states that the current is proportional to the voltage across a conductor. The ratio

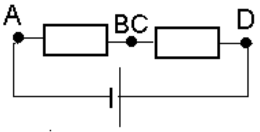
$$R = \frac{V}{I}$$

is called resistance. The unit of resistance is the Ohm (Ω).

Connecting devices in series and in parallel; resultant resistance

Series connection





Two resistors or other devices, elements of the circuit are connected ***in series*** when they are joined at one end but no other element is connected to their joining point. The free ends of the joined elements are connected into the circuit.
The same current flows through each element in the whole chain as there is no branching point:

$I_{R1} = I_{R2} = I.$

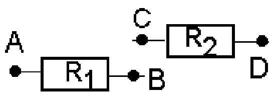
The voltages across the resistors add up:

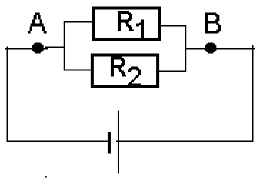
$V_{AD} = V_{R1} + V_{R2}$.

The chain of resistors between A and D can be replaced by a single one with a resistance equal to the series resultant R_s of the individual resistances. This series resultant can be calculated knowing that the voltages add up and the same current flows through both resistors, so

$$V_{AD} = V_{R1} + V_{R2} \quad \rightarrow \quad I \cdot R_s = I \cdot R_1 + I \cdot R_2 \quad \rightarrow \quad R_s = R_1 + R_2$$

Parallel connection





Two resistors or other devices are connected in parallel if they are joined at both ends and there is no other element on the connecting wires.
The voltage is the same across both resistors as it is the difference between the potentials of that of the connecting wires:

$V_{R1} = V_{R2} = V_{AB}$.

The current flowing through the branches adds up:

$I_{AB} = I_{R1} + I_{R2}$.

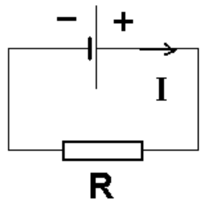
The joined resistors can be replaced by a single one with a resistance equal to the parallel resultant R_p. The current through this resultant equals the current flowing out from and back into the source. This means that the sum of the current intensities flowing through both resistors is equal to the current flowing through the source.
Combining the additivity of currents with Ohm's law, we get the resultant resistance:

$$I = I_{R1} + I_{R2} \quad \rightarrow \quad \frac{V_{AB}}{R_p} = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2} \quad \rightarrow \quad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Sources of electricity: galvanic cells, batteries, generators

There are several means to produce electrical energy from other types of energy.
In a power plant, the generators convert mechanical energy into electric energy. The generators gain their mechanical energy from heat, or from the energy of wind or flowing waters. The commonly used galvanic cells and batteries convert chemical energy into electricity. The solar cells convert light energy – the energy of sunshine into electric energy.

A galvanic cell consists of two electrodes (metals or metal and carbon) immersed into some electrolyte. The ions of metals tend to dissolve in the electrolyte, leaving their outer electrons behind and making this electrode negative with respect to the electrolyte. Different metals dissolve more or less readily so there will be a potential difference between the electrodes. The magnitude of the potential difference between the electrodes of a galvanic cell is called the ***electromotive force ε (emf)*** of the cell. Connecting the electrodes by a metal wire, the electrons accumulated on the negative electrode start to flow toward the positive electrode: we get current in the wire the direction of which is just opposite to the drift velocity of the electrons.



To use the electrical energy accumulated in the cell, we connect some load to it: a light bulb, for example. The source and the load connected to it make the simplest electric circuit. The load is represented by a resistor, with resistance R. The electric current flows always from + to – across a resistor. According to Ohm's law, the voltage across the resistor is U = RI. This voltage usually is lower than the emf of the cell, as the cell itself has got some internal resistance, R_i. As the current flows around, it has to flow across the internal resistance, producing a voltage drop there. So we get Ohm's law for the whole circuit as

$$\varepsilon = I(R + R_i).$$

The voltage across the terminals of a loaded source is called **terminal voltage**, V_T.

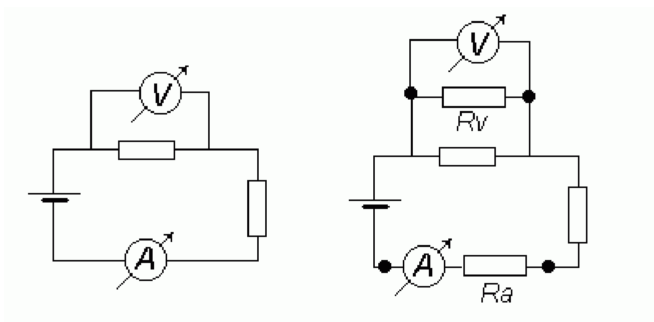
$$V_T = \varepsilon - R_i I.$$

Connecting meters into the circuit

Voltmeters measure voltage <u>across two points</u> of a circuit, so they are connected <u>in parallel</u> to the points where we want to know the voltage.

Ammeters measure current <u>in a branch</u> of the circuit, so they are connected <u>in series</u> with the elements where we want to know the current (we have to break the circuit to insert the ammeter).

Ideal voltmeters have infinite resistance, ideal ammeters have zero resistance, but no meters are ideal so the current and voltage they measure will differ somewhat from the ones without the meter. To calculate the effect of the meter to currents and voltages in the circuit we replace the meter by its internal resistance in the circuit-diagram. The voltmeter reading then is equal to the voltage across its internal resistance. The reading of the ammeter is equal to the current flowing through its internal resistance.



Placing voltmeter and ammeter in a circuit and the effect of their internal resistance