

OPTICS 1: GEOMETRICAL OPTICS

THEORY¹

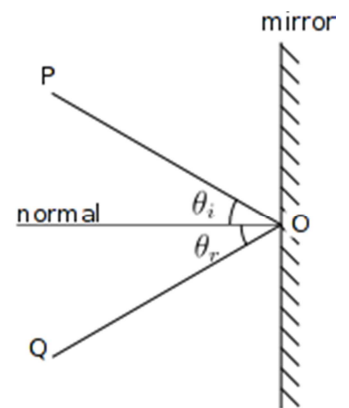
Geometrical optics, or ray optics, describes light propagation in terms of "rays". The "ray" in geometric optics is an abstraction, or "instrument", which can be used to approximately model how light will propagate. Light rays bend at the interface between two dissimilar media, and may curve in a medium where the refractive index changes. Geometrical optics provides rules for propagating these rays through an optical system. The path taken by the rays indicates how the actual wave will propagate. This is a significant simplification of optics that fails to account for optical effects such as diffraction and polarization. It is a good approximation, however, when the wavelength is very small compared with the size of structures with which the light interacts. Geometrical optics can be used to describe the geometrical aspects of imaging, including optical aberrations.

Reflection

Glossy surfaces such as mirrors reflect light in a simple, predictable way. This allows for production of reflected images that can be associated with an actual (real) or extrapolated (virtual) location in space.

With such surfaces, the direction of the reflected ray is determined by the angle the incident ray makes with the surface normal, a line perpendicular to the surface at the point where the ray hits. The incident and reflected rays lie in a single plane, and the angle between the reflected ray and the surface normal is the same as that between the incident ray and the normal. This is known as the *Law of Reflection*.

Mirrors with curved surfaces can be modeled by ray tracing and using the law of reflection at each point on the surface. For mirrors with parabolic surfaces, parallel rays incident on the mirror produce reflected rays that converge at a common focus. Other curved surfaces may also focus light, but with aberrations due to the diverging shape causing the focus to be smeared out in space. In particular, spherical mirrors exhibit spherical aberration. Curved mirrors can form images with magnification greater than or less than one, and the image can be upright or inverted. An upright image formed by reflection in a mirror is always virtual, while an inverted image is real and can be projected onto a screen.

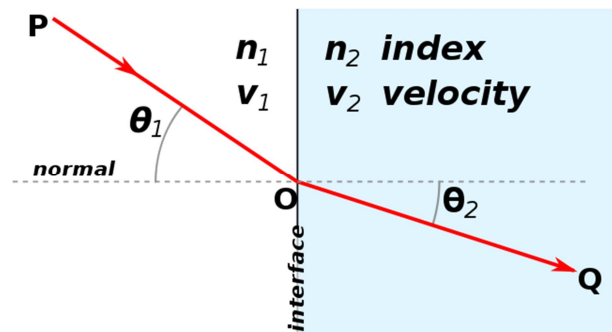


Refraction

Refraction occurs when light travels through an area of space that has a changing index of refraction. The simplest case of refraction occurs when there is an interface between a uniform medium with index of refraction n_1 and another medium with index of refraction n_2 . In such situations, **Snell's Law** describes the resulting deflection of the light ray:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad [F1]$$

where θ_1 and θ_2 are the angles between the normal (to the interface) and the incident and refracted waves, respectively.

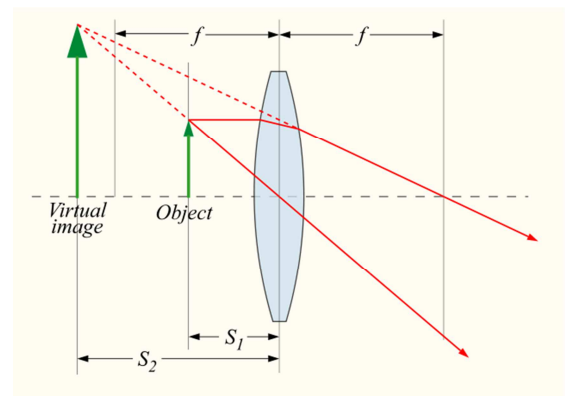


¹ The paragraphs and images were copied from Wikipedia: <http://en.wikipedia.org/wiki/Optics>,
http://en.wikipedia.org/wiki/Geometrical_optics

Various consequences of Snell's Law include the fact that for light rays traveling from a material with a high index of refraction to a material with a low index of refraction, it is possible for the interaction with the interface to result in zero transmission. This phenomenon is called **total internal reflection** and allows for fiber optics technology. As light signals travel down a fiber optic cable, it undergoes total internal reflection allowing for essentially no light lost over the length of the cable. It is also possible to produce polarized light rays using a combination of reflection and refraction: When a refracted ray and the reflected ray form a right angle, the reflected ray has the property of "plane polarization". The angle of incidence required for such a scenario is known as **Brewster's angle**.

Snell's Law can be used to predict the deflection of light rays as they pass through "linear media" as long as the indexes of refraction and the geometry of the media are known. For example, the propagation of light through a **prism** results in the light ray being deflected depending on the shape and orientation of the prism. Additionally, since different frequencies of light have slightly different indexes of refraction in most materials, refraction can be used to produce **dispersion spectra** that appear as rainbows.

A device which produces converging or diverging light rays due to refraction is known as a **lens**. Thin lenses produce focal points on either side that can be modeled using the lensmaker's equation. In general, two types of lenses exist: convex lenses, which cause parallel light rays to converge, and concave lenses, which cause parallel light rays to diverge. The detailed prediction of how images are produced by these lenses can be made using ray-tracing similar to curved mirrors. Similarly to curved mirrors, thin lenses follow a simple equation that determines the location of the images given a particular **focal length (f)** and **object distance ($d_{obj} = S_1$)**:



$$\frac{1}{d_{obj}} + \frac{1}{d_{img}} = \frac{1}{f} \quad [F2]$$

where **$d_{img} = S_2$ is the distance associated with the image** and is considered by convention to be negative if on the same side of the lens as the object and positive if on the opposite side of the lens. The focal length f is considered negative for concave lenses.

Incoming parallel rays are focused by a convex lens into an inverted real image one focal length from the lens, on the far side of the lens. Rays from an object at finite distance are focused further from the lens than the focal distance; the closer the object is to the lens, the further the image is from the lens. With convex lenses, incoming parallel rays diverge after going through the lens, in such a way that they seem to have originated at an upright virtual image one focal length from the lens, on the same side of the lens that the parallel rays are approaching on. Rays from an object at finite distance are associated with a virtual image that is closer to the lens than the focal length, and on the same side of the lens as the object. The closer the object is to the lens, the closer the virtual image is to the lens.

Likewise, the **magnification M** of a lens is given by

$$M = -\frac{d_{img}}{d_{obj}} = -\frac{s_{img}}{s_{obj}} \quad [F3]$$

where **s_{obj} and s_{img} are the size of the object and the image**, respectively, and the negative sign

is given, by convention, to indicate an upright object for positive values and an inverted object for negative values. Similar to mirrors, upright images produced by single lenses are virtual while inverted images are real.

Lenses suffer from aberrations that distort images and focal points. These are due to both to geometrical imperfections and due to the changing index of refraction for different wavelengths of light (chromatic aberration).

MEASUREMENTS

1. Determination of the focal length of a convex lens

Tools:

- optical bench
- halogen lamp light source
- slide holder
- a slide as object
- convex lens
- screen

Place the light source on one end of the optical bench and the screen on the other end. The teacher tells you the distance between the object and the screen. Place the object – i.e. the slide in the slide holder – accordingly between the lamp and the screen. Finally, the lens is placed between the object and the screen. Then by sliding the lens find that position or those positions, where a sharp image of the object appears on the screen. Measure the distance between the object and the lens d_{obj} and the distance between the image and the lens d_{img} , and measure (or estimate) the size of the image s_{img} .

Evaluation:

- 1/1. Calculate the focal length of the lens using formula [F2].
- 1/2. Calculate the error interval for the focal length using the error propagation formula. Take 1 cm as the error of distance measurement for d_{obj} and d_{img} .
- 1/3. Draw a scaled diagram of the arrangement showing the most important rays.
- 1/4. Calculate the size of the object using formula [F3].

2. Estimation of the thickness of a hair fiber using a lens

Tools:

- optical bench
- halogen lamp light source
- slide holder
- hair fiber in a slide
- convex lens, $f = 50$ mm
- screen

Place the light source on one end of the optical bench, the screen on the other end and in between the slide holder with the hair fiber and the lens. By sliding the lens find the largest possible sharp image. Measure the object distance d_{obj} and image distance d_{img} , and measure / estimate the thickness of the image of the hair fiber on the screen s_{img} .

Evaluation:

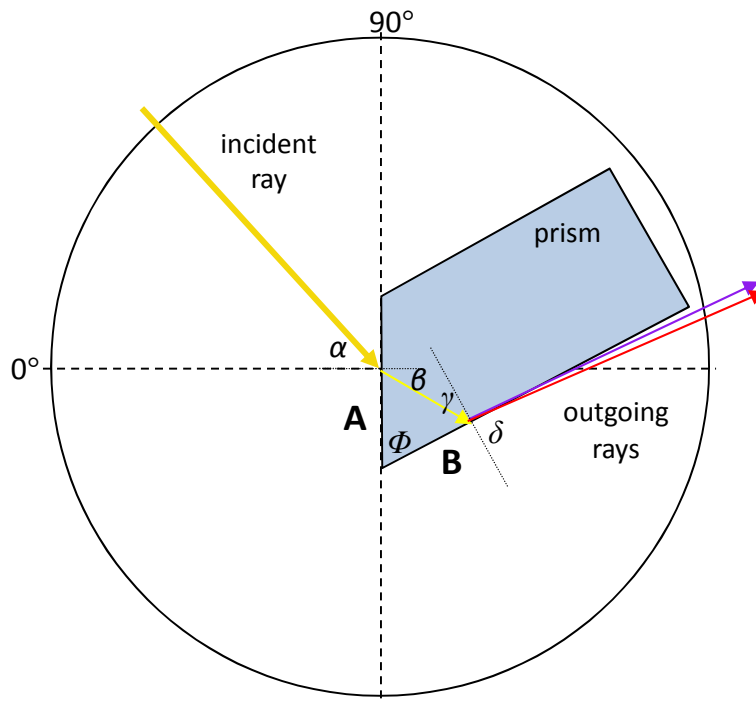
- 2/1. Calculate the magnification, and use it to
- 2/2. ... "calculate" the thickness of a human hair. (Is this measurement accurate? why?)

3. Determination of the refractive index of prism

Tools:

- optical bench
- halogen lamp light source
- gaps
- slide holder
- rotating optical disc
- prism

Measure the angle of incidence α when $\delta = 90^\circ$.



Evaluation:

Derivation of the formula:

We know that

at the first interface $\sin \alpha = n \sin \beta$, at the second interface $n \sin \gamma = \sin \delta$, and from the geometry $\Phi = \beta + \gamma$.

For $\delta = 90^\circ$ $\sin \delta = \sin 90^\circ = 1 \rightarrow n \sin \gamma = 1$;

and as $\gamma = \Phi - \beta$: $n \sin \gamma = n \sin(\Phi - \beta) = n(\sin \Phi \cos \beta - \cos \Phi \sin \beta) = n \sin \Phi \cos \beta - n \cos \Phi \sin \beta$.

Substituting $n \sin \beta = \sin \alpha$: $n \sin \Phi \cos \beta - \cos \Phi \sin \alpha = 1 \rightarrow n \cos \beta = (1 + \cos \Phi \sin \alpha) / \sin \Phi$.

Adding the square of the last equation and the square of the equation $n \sin \beta = \sin \alpha$ we get

$$n^2 = (1 + 2 \cos \Phi \sin \alpha + \cos^2 \Phi \sin^2 \alpha) / \sin^2 \Phi + \sin^2 \alpha = (1 + 2 \cos \Phi \sin \alpha + \sin^2 \alpha) / \sin^2 \Phi,$$

and expressing n we get the refractive index of the prism as a function of the angle of incidence α :

$$n = \sqrt{\frac{1 + 2 \cos \Phi \sin \alpha + \sin^2 \alpha}{\sin^2 \Phi}} ; \quad \text{in our case } \Phi = 60^\circ.$$

3/1. Calculate the refractive index of the prism for the red and for the violet component.

3/2. Take $\Delta \alpha = 1^\circ$ as the error of the angle of incidence, and calculate the error of the refractive index using the error propagation formula.