

## The simple pendulum

### THEORY

The equation of motion of a small object  $m$  suspended from the end of a light, inextensible string of length  $L$  swinging in a vertical plane is the following:

$$m \cdot a_t = m \cdot L \frac{d^2 \alpha}{dt^2} = -m \cdot g \cdot \sin \alpha$$

where  $\alpha$  is the time dependent angle of the string measured from the equilibrium (vertical) position. As it is a nonlinear differential equation we introduce the following approximation:

$$\sin \alpha \approx \alpha$$

and so the problem is similar to that of the spring:

$$m \cdot L \frac{d^2 \alpha}{dt^2} = -m \cdot g \cdot \alpha \quad \rightarrow \quad \frac{d^2 \alpha}{dt^2} = -(g/L) \cdot \alpha$$

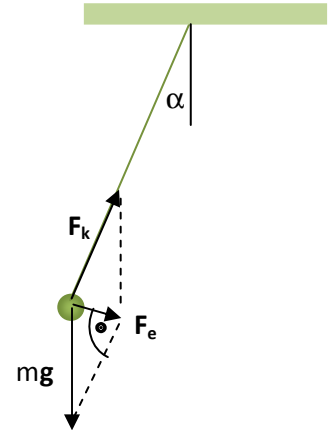
$$\alpha(t) = \alpha_{\max} \cos(\omega t + \varphi_0) \quad \rightarrow \quad \frac{d^2 \alpha}{dt^2} = -\omega^2 \cdot \alpha$$

$$\rightarrow \quad \omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{T}$$

This means that the motion of the pendulum is harmonic with a time period of

$$T = 2\pi \sqrt{\frac{L}{g}}.$$

The deviation caused by the approximation  $\sin \alpha \approx \alpha$  is 0.05% for  $\alpha=5^\circ$ , 1% for  $\alpha=22^\circ$  and 18% for  $\alpha=90^\circ$ .



### MEASUREMENT

Set the pendulum bob in motion with a small initial angle.

Measure the time of 10 periods.

Repeat the measurement 5 times.

Measure the length of the string  $L$ .

Measure the time of 10 periods when the initial angle is increased.

### EVALUATION

Calculate the average time period  $\bar{T}$  and the confidence interval  $\Delta T$  for  $P = 95\%$ .

Calculate the acceleration of gravity  $g$ .

Estimate the error of measuring the length of the string  $\Delta L$ .

Calculate the error of  $g$  using the formula of error propagation.