The simple pendulum

THEORY

The equation of motion of a small object *m* suspended from the end of a light, inextensible string of length *L* swinging in a vertical plane is the following:

$$m \cdot a_t = m \cdot L \frac{d^2\alpha}{d\,t^2} = -\,m \cdot g \cdot \sin\alpha$$

where α is the time dependent angle of the string measured from the equilibrium (vertical) position. As it is a nonlinear differential equation we introduce the following approximation:

$$\sin\alpha \approx \alpha$$

and so the problem is similar to that of the spring:

$$m \cdot L \frac{d^2\alpha}{d\,t^2} = -\,m \cdot g \cdot \alpha \quad \rightarrow \quad \frac{d^2\alpha}{d\,t^2} = -(g/L) \cdot \alpha$$

$$\alpha(t) = \alpha_{max} \cos(\omega t + \varphi_0) \quad \rightarrow \quad \frac{d^2\alpha}{d\,t^2} = -\omega^2 \cdot \alpha$$

$$\rightarrow \qquad \omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{T}$$

This means that the motion of the pendulum is harmonic with a time period of

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}\,.$$

The deviation caused by the approximation $\sin\alpha \approx \alpha$ is
0.05% for α=5°, 1% for α=22° and 18% for α=90°.

MEASUREMENT

Set the pendulum bob in motion with a small initial angle.
Measure the time of 10 periods.
Repeat the measurement 5 times.
Measure the length of the string *L*.
Measure the time of 10 periods when the initial angle is increased.

EVALUATION

Calculate the average time period $\bar{T}$ and the confidence interval ΔT for P = 95%.
Calculate the acceleration of gravity *g*.
Estimate the error of measuring the length of the string ΔL.
Calculate the error of *g* using the formula of error propagation.